The Calculation of Thermal Conductivity for Nanofluids Containing Nanoparticles and Carbon Nanotubes

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Abstract

The paper features the mathematical model of calculation of thermal conductivity for nanofluids containing nanoparticles and nanotubes on the basis of statistical mechanics. Calculation of transport properties for nanofluids for real substances is possible by the classical and statistical mechanics. Classical mechanics has no insight into the microstructure of the substance. Statistical mechanics, on the other hand, calculates the properties of state on the basis of molecular motions in a space, and on the basis of the intermolecular interactions. The equations obtained by means of classical thermomechanics are empirical and apply only in the region under observation.

In the presented paper we have focused on the influence of liquid layer structure around nanoparticles as the reason of thermal conductivity enhancement of nanofluids. The results of the analysis are compared with experimental data obtained in the scientific literature and show a relatively good agreement. The analytical results obtained by statistical mechanics are compared with the experimental data and the comparison shows relatively good agreement.
I. INTRODUCTION

Only a decade or two ago, many renowned experts believed that models and theories of classical mechanical engineering science had in principle been developed and that any further evolution options were completely exhausted. In other words, engineers could only do some “fine grinding” and model adjustment in the future. Fortunately, those experts were totally wrong. Let us enumerate some of the main innovations in modelling or in the application of physics supported by mathematics to engineering practice.

1. In mechanics in general, chaos theory has brought about a revolution not seen in mechanical engineering for many years. Nonlinear elements in equations suddenly became extremely important. So far, the majority of problems in the field of nonlinear mechanics have been explained and anticipated by means of chaos theory, whereas certain issues, such as turbulence, have not yet been successfully resolved.

2. Nanomechanics and micromechanics are becoming increasingly important in today’s industry. The concepts of invisible aircraft, pumps … are now a reality. At the same time, problems have arisen in advanced mechanics not even dreamed of before. Thermodynamic and transport properties of a gas flowing through a tube with the diameter of a few nanometres are modelled completely differently due to a great influence of surface effects. Even classical hydromechanics is not of much help here. In addition to temperature and pressure the Knudsen number is becoming increasingly important. Euler’s equation gives bad results almost over the entire range, Navier-Stokes’ equation at Knudsen number 0.1 and Burnett’s equation at Knudsen number 10. However, in order to analyse free molecular flow in micro and nanochannels the non-equilibrium mechanics and the original Boltzmann’s equation have to be used. In this
case, computation of hydromechanical problems is possible over the entire range of Knudsen, temperatures and pressures.

3. The economic importance of nanofluids is growing due to their exceptional properties. For the time being, empirical mixing rules apply in nanoparticles-fluid mixture modelling which, however, do not deal with the core of the problem. In addition to nanofluids, the application of nanotubes with even better properties is increasing in the engineering practice.

4. Non-equilibrium mechanics will bring about revolution in the field of non-equilibrium phenomena modelling. Today, the theory is not completely resolved. Boltzmann’s theory provides a basis for it, yet no analytical procedures have been developed for multi-atom molecules and high density systems. In the future, the theory of non-equilibrium mechanics will help resolve the problems of the mechanics of fluids and thermodynamics in mechanical engineering in a completely different way than today.

5. The theory of genetics in the DNA level is at this time the most important science. On this basis are created now the new science disciplines important for engineering such as biomechanics, nano-biomechanics, mechanics of living organisms.

6. When the theory of gravity will be completely solved the application in engineering will be of enormous importance.

The term nanofluid is envisioned to describe a solid-liquid mixture which consists of a nanoparticles and a base liquid and this is one of new challenges for thermo-sciences provided by the nano-technology. The possible application area of nanofluids is in advanced cooling systems, in micro/nano electromechanical systems… The investigation of the effective thermal conductivity of liquid with nanoparticles attract much more interest experimentally and
theoretically. The effective thermal conductivity of nanoparticle suspension can be much higher than for the fluid without nanoparticles.

Calculation of transport properties for nanofluids for real substances is possible by the classical and statistical mechanics. Classical mechanics has no insight into the microstructure of the substance. Statistical mechanics, on the other hand, calculates the properties of state on the basis of molecular motions in a space, and on the basis of the intermolecular interactions. The equations obtained by means of classical thermodynamics are empirical and apply only in the region under observation. The main drawback of classical thermodynamics is that it lacks the insight into the substance of microstructure. Contrary to classical mechanics, statistical mechanics calculates the thermomechanic properties of state on the basis of intermolecular and intramolecular interactions between particles in the same system of molecules. It deals with the systems composed of a very large number of particles.

In this paper are determined new constants for fluids. The results of the analysis are compared with experimental data and show a relatively good agreement.

II. CALCULATION OF THERMAL CONDUCTIVITY FOR PURE FLUID [1-18]

Accurate knowledge of nonequilibrium or transport properties of pure gases and liquids, is essential for the optimum design of the different items of chemical process plants, for determination of intermolecular potential energy functions and for development of accurate theories of transport properties in dense fluids. Transport coefficients describe the process of relaxation to equilibrium from a state perturbed by application of temperature, pressure, density, velocity or composition gradients. The theoretical description of these phenomena constitutes that part of nonequilibrium statistical mechanics that is known as kinetic theory.
In the presented paper will be presented Chung-Lee-Starling model (CLS) [14-16]. Equations for the thermal conductivity are developed based on kinetic gas theories and correlated with the experimental data. The low-pressure transport properties are extended to fluids at high densities by introducing empirically correlated, density dependent functions. These correlations use acentric factor $\omega$, dimensionless dipole moment $\mu$, and empirically determined association parameters to characterize molecular structure effect of polyatomic molecules $\kappa$, the polar effect and the hydrogen bonding effect. In this paper are determined new constants for fluids.

The dilute gas thermal conductivity for CLS model is written as:

$$\lambda = \lambda_k + \lambda_p$$  \hspace{1cm} (1)

where:

$$\lambda_k = \lambda_0 \left( \frac{1}{H_2} + B_6 Y \right)$$  \hspace{1cm} (2)

The thermal conductivity in the region of dilute gases for CLS model is written as:

$$\lambda_0 = 3119.41 \left( \frac{\eta_0}{M} \right)^\psi$$  \hspace{1cm} (3)

where $\psi$ represents the influence of polyatomic energy contributions to the thermal conductivity.

We used the Taxman theory [2]. He solved the problem of influence of internal degrees of freedom on the basis of WCUB theory [1-4] and the approximations given by Mason and Monschick [2-4]. The final expression for the influence of internal degrees of freedom is represented as:
\[
\psi = 1 + C^*_{\text{int}} \left\{ 0.2665 \frac{(0.215 - 1.061\beta)}{Z_{\text{coll}}} + 0.28288 \frac{C^*_{\text{int}}}{Z_{\text{coll}}} \right\} \\
\beta + \frac{0.6366}{Z_{\text{coll}}} + \frac{1.061\beta C^*_{\text{int}}}{Z_{\text{coll}}} \right\}
\]

where \( C^*_{\text{int}} \) is the reduced internal heat capacity at constant volume, \( \beta \) is diffusion term and \( Z_{\text{coll}} \) is the collision number. The heat capacities are calculated by use of statistical thermodynamics.

The paper features all important contributions (translation, rotation, internal rotation, vibration, intermolecular potential energy and influence of electron and nuclei excitation). The residual part \( \lambda_p \) to the thermal conductivity can be represented with the following equation

\[
\lambda_p = \left( 0.1272 \left( \frac{T_c}{M} \right)^{1/2} \frac{1}{V_{c}^{2/3}} \right) B_7 Y^2 H_2 \left( \frac{T}{T_c} \right)^{1/2}
\]

where \( \lambda_p \) is in W/mK.

\[
H_2 = \left\{ B_1 \left[ 1 - \exp(-B_4 Y) \right] \frac{1}{Y} + B_2 G_1 \exp(B_3 Y) + B_3 G_1 \right\} \frac{1}{B_1 B_4 + B_2 + B_3}
\]

The constants \( B_1-B_7 \) are linear functions of acentric factor, reduced dipole moment and the association factor

\[
B_i = b_0(i) + b_1(i)\omega + b_2(i)\mu_+^4 + b_3(i)\kappa \quad i=1,10
\]

where the coefficients \( b_0, b_1, b_2 \) and \( b_3 \) are presented in the work of Chung et al. [14-16]
III. THE CALCULATION OF THERMAL CONDUCTIVITY FOR PURE SOLIDS

[19-28]

3.1 Electronic contribution to the thermal conductivity

The fundamental expression for electronic contribution \( \lambda_{el} \) to the thermal conductivity can be calculated on the basis of the theory of thermal conductivity for classical gas:

\[
\lambda_{el} = \frac{1}{3} n c_{el} v_{el} l_{el}
\]

(8)

where \( c_{el} \) is the electronic heat capacity (per electron), \( n \) is the number of conduction electrons per volume, \( v_{el} \) is the electron speed and \( l_{el} \) is the electron mean free path. In the equation (8) it is assumed that in temperature gradient electrons travel just the same average distance \( l \) before transferring their excess thermal energy to the atoms by collisions.

We can express the mean free path with help of electron lifetime \( \tau \) (\( l_{el} = v_F \tau \)):

\[
\lambda_{el} = \frac{\pi^2 n k_B \tau}{3m}
\]

(9)

With help of Drude theory [22,23] we can express thermal conductivity as the function of electrical conductivity \( \sigma \):

\[
\lambda_{el} = \sigma L T
\]

(10)

where \( L \) is temperature dependent constant.

3.2 Phonon contribution to the thermal conductivity

It is more difficult to determine the thermal conductivity when there are non-free electrons. Solids which obey this rule we called non-metallic crystals. Because the atoms in a solid are closely coupled together, an increase in temperature, will be transmitted to the other parts. In the
modern theory, heat is being considered as being transmitted by phonons, which are the quanta of energy in each mode of vibration. We can again use the expression:

\[ \lambda_{ph} = \frac{1}{3} Cv \]

(11)

3.3 The calculation of electronic contribution using Eliashberg transport coupling function

In the book of Grimwall [22] we can find the analytical expression for the electrical conductivity \( \sigma \):

\[ \sigma_e = \frac{ne^2}{m_b} \left\langle \tau(\varepsilon, \vec{k}) \right\rangle \]

(12)

In the Equation (7) \( m_b \) represents electron band mass and \( \tau \) is an electron lifetime that depends both on the direction of the wave vector \( \vec{k} \) and on the energy distance \( \varepsilon \). The brackets \( \left\langle . \right\rangle \) describe an average over all electron states. We can also describe the electronic part of thermal conductivity with help of Eq. (12):

\[ \lambda_{el} = \frac{nk_BT}{m_b} \left\langle \left( \frac{\varepsilon_k - E_F}{k_BT} \right)^2 \tau(\varepsilon, \vec{k}) \right\rangle \]

(13)

The lifetime for the scattering of electrons by phonons contains quantum-mechanical quantum matrix elements for the electron-phonon interaction and statistical Bose-Einstein and Fermi-Dirac factors for the population of phonon and electron states. A very useful magnitude in the context is the Eliashberg transport coupling function \( \alpha^2 F(\omega) \). A detailed theoretical expression is possibly to find in the work of Grimwall [23]. The Eliashberg coupling function allows us to write the thermal conductivity in the next expression:
\[
\frac{1}{\lambda_{el}} = \frac{(4\pi)^2}{L_0 T_0 \omega^2} \int_0^{\omega_{pl}} \frac{h\omega / k_B T}{[\exp(h\omega / k_B T - 1)][1 - \exp(-h\omega / k_B T)]} \left[ 1 - \frac{1}{2\pi^2} \left( \frac{h\omega}{k_B T} \right)^2 \right] \alpha_{tr}^2 F(\omega) \, d\omega \tag{14}
\]

We can describe the phonons by an Einstein model

\[
\alpha_{tr}^2 F(\omega) = A \delta(\omega - \omega_E) \tag{15}
\]

\[
\alpha^2 F(\omega) = B \delta(\omega - \omega_E) \tag{16}
\]

In the equations (15) and (16) are B and A constants. With help of equations (15) and (16) we can solve integral in equation (14):

\[
\frac{1}{\lambda_{el}} = k_E C_{har} \left( \frac{T}{\theta_E} \right) \left[ \frac{A}{B} + \left( \frac{\theta_E}{T} \right)^2 \frac{1}{2\pi^2} \left( 3 - \frac{A}{B} \right) \right] \tag{17}
\]

In the equation (17) \( k_E \) represents the constant, \( \theta_E \) is the Einstein temperature and \( C_{har} \) represents the lattice heat capacity in Einstein model:

\[
C_{har} = 3Nk_B T \left( \frac{\theta_E}{T} \right)^2 \frac{\exp \left( \frac{\theta_E}{T} \right)}{\left( \exp \left( \frac{\theta_E}{T} \right) - 1 \right)^2} \tag{18}
\]

Motokabbir and Grimwall [11] discussed about equation (17) with \( A/B \) as a free parameter with assumption that \( A/B \approx 1 \).

### 3.4 The phonon contribution to thermal conductivity

In an isotropic solid we can express the thermal conductivity as the integral over \( \omega \) containing the phonon density of states \( F(\omega) \) [9]:
\[ \lambda_{ph} = \frac{N}{3V} v_g^2 \int_0^{\omega_{max}} \tau(\omega)C(\omega)F(\omega)\,d\omega \quad (19) \]

where \( v_g \) is some average phonon group velocity, \( C \) is the heat capacity of a single phonon mode and the ratio \( N/V \) is the number of atoms per volume.

A relaxation time can be expressed as the ratio of a mean free path to a velocity, so that the thermal conductivity can be expressed as:

\[ \lambda_{ph} = \frac{N}{3V} v_g^2 \int_0^{\omega_{max}} \tau(\omega)C(\omega)F(\omega)\,d\omega \quad (20) \]

The crucial point in equation (20) is the determination of relaxation time. If we consider scattering in and out of state 1 we can with help of quantum mechanics describe \( \tau(1) \):

\[ \frac{1}{\tau(1)} = \frac{2\pi}{\hbar} \sum_{2,3} |H(1,2,3)|^2 \frac{n(2)n(3)}{n(1)} \quad (21) \]

\[ |H(1,2,3)|^2 = \frac{A}{3MN} \frac{\omega_1 \omega_2 \omega_3}{v_g^2} \quad (22) \]

The evaluation of \( \tau(1) \) in equation (21) requires a summation over modes 2 and 3. This cannot be done analytically, so it is not possible to give a closed-form expression for the temperature dependence of the thermal conductivity valid at all temperatures.

For the low temperature region (where the temperature is lower than Debye temperature \( \theta_D \)), we have used the solution:

\[ \lambda_{ph} = \lambda_0 \exp\left( -\frac{\theta_D}{T} \right) \quad (23) \]

where \( \lambda_0 \) is the constant.
For the high temperature region \( T >> \theta_D \) the solution of Eq. (23) gives the result:

\[
\lambda_{ph} = \frac{B \cdot M \Omega_a^{1/3} k_B \theta_D^3}{(2\pi)^3} \frac{\theta}{h^3 \gamma^2 T} \tag{24}
\]

where \( B \) is dimensionless constant, \( \Omega_a \) is atomic volume and \( \gamma \) is the Grüneisen constant. The relation between the Einstein and Debye temperature may be written as:

\[
\theta_E = (0.72..0.75) \theta_D \tag{25}
\]

IV. THE CALCULATION OF THERMAL CONDUCTIVITY FOR NANO PARTICLES [29-37]

In nanoparticle fluid mixtures, other effects such as microscopic motion of particles, particle structures and surface properties may cause additional heat transfer in nanofluids. Nanofluids also exhibit superior heat transfer characteristics to conventional heat transfer fluids. One of the main reasons is that suspended particles remarkably increase thermal conductivity of nanofluids. The thermal conductivity of nanofluid is strongly dependent on the nano-particle volume fraction. So far it has been an unsolved problem to develop a sophisticated theory to predict thermal conductivity of nanofluids. The presented paper is the attempt to calculate thermal conductivity of nanofluid analytically. Hamilton and Crosser developed the model for the effective thermal conductivity of two-component mixtures as a function of the conductivity of the pure materials, the composition and shape of dispersed particles. The thermal conductivity can be calculated then with the next expression [29-34]:

\[
\lambda = \lambda_0 \left[ \frac{\lambda_p + (n-1)\lambda_0 - (n-1)\alpha(\lambda_0 - \lambda_p)}{\lambda_p + (n-1)\lambda_0 + \alpha(\lambda_0 - \lambda_p)} \right] \tag{26}
\]
where \( \lambda \) is the mixture thermal conductivity, \( \lambda_0 \) is the liquid thermal conductivity, \( \lambda_p \) is the thermal conductivity of solid particles, \( \alpha \) is the volume fraction and \( n \) is the empirical shape factor given by,

\[
\alpha = \frac{V_p}{V_0 + V_p} = n \frac{\pi}{6} d_p^3
\]

(28)

where \( \psi \) is sphericity, defined as the ratio of the surface area of a sphere (with a volume equal to that of a particle) to the area of the particle.. The volume fraction \( \alpha \) of the particles is defined as:

\[
\alpha = \frac{V_p}{V_0 + V_p} = n \frac{\pi}{6} d_p^3
\]

(28)

where \( n \) is the number of the particles per unit volume and \( d_p \) is the average diameter of particles. An alternative expression for calculating the effective thermal conductivity of solid-liquid mixtures was introduced by Wasp:

\[
\lambda = \lambda_0 \left[ \frac{\lambda_p + 2\lambda_0 - 2\alpha(\lambda_0 - \lambda_p)}{\lambda_p + 2\lambda_0 + \alpha(\lambda_0 - \lambda_p)} \right]
\]

(29)

Comparison between Eq. (26) and Eq. (29) shows that Wasp model is a special case with the sphericity of 1.0 of the Hamilton and Crosser model. From the literature [31-34] we can find some other models (Maxwell, Jeffrey, Davis, Lu-Lin..) with almost identical analytical results.

The HC model gives very good results for particles larger than 13 nm. For smaller particles the presented theory gives wrong results with the deviation more than 100% in comparison with experimental results. The presented theoretical models for the calculation of the thermal conductivity for nanofluids are only dependent on the thermal conductivity of the solid and the
liquid and their relative volume fraction, but not on particle size and the interface between particles and the fluid.

In the scientific literature we can find many reasons of thermal conductivity enhancement in nanofluids. In the presented paper we have made the assumption that the thermal conductivity enhancement in nanofluids is due to formation of liquid layers around of nanoparticles.

For the calculation of effective thermal conductivity we have used Xue theory [34], based on Maxwell theory and average polarization theory. Because the interfacial shells are existed between the nanoparticles and the liquid matrix, we can regard both the interfacial shell and the nanoparticle as a complex nanoparticle. So the nanofluid system should be regarded as the complex nanoparticles dispersed in the fluid. We assume that $\lambda$ is the effective thermal conductivity of the nanofluid, $\lambda_{c}$ and $\lambda_{m}$ are the thermal conductivity of the complex nanoparticles and the fluid, respectively. The final expression of Xue model (X) is expressed with the next equation:

\[
9 \left(1 - \frac{\alpha}{\lambda_r}\right) \frac{\lambda_c - \lambda_{0}}{2\lambda + \lambda_{0}} + \frac{\alpha}{\lambda_r} \left[ \frac{\lambda - \lambda_{c,x}}{\lambda + B_{2x}\left(\lambda_{c,x} - \lambda_c\right)} + 4 \frac{\lambda - \lambda_{c,y}}{2\lambda + (1 - B_{2x})\left(\lambda_{c,y} - \lambda_c\right)} \right] = 0
\]  

(30)

\[
\lambda_{c,j} = \frac{\lambda_2 \left(1 - B_{2,j}\right)\lambda_{1} + B_{2,j}\lambda_{2} + \left(1 - B_{2,j}\right)\lambda_{j}(\lambda_{2} - \lambda_{1})}{\left(1 - B_{2,j}\right)\lambda_{1} + B_{2,j}(\lambda_{2} - B_{2,j}\lambda_{1})(\lambda_{2} - \lambda_{1})}
\]  

(31)

We assume that the complex nanoparticle is composed of an elliptical nanoparticle with thermal conductivity $\lambda_{2}$ with halfradii of (a,b,c) and an elliptical shell of thermal conductivity $\lambda_{1}$ with a thickness of t. In Eq. (30) and Eq. (31) $\lambda_{1}$ represents the spatial average of heat flux component. For simplicity we assume that all fluid particles are balls and all the nanoparticles are the same rotational ellipsoid.
Many studies have focused on the effect of the liquid layer, we considered nanoparticle-in-liquid suspension with monosized spherical particles of radius $r$ and particle volume concentrations $\alpha$. We have used the model of Yu and Choi [38] that the nanolayer of each particle could be combined with the particle to form an equivalent particle and that the particle volume concentration is so low that there is no overlap of those equivalent particles. On this basis we can express the effective volume fraction:

$$\alpha_e = \alpha \left(1 + \frac{h}{r}\right)^3,$$  \hspace{1cm} (32)

where $h$ represents the liquid layer thickness. We have also made the assumption that equivalent thermal conductivity of the equivalent particles has the same value as the thermal conductivity of particle. On the basis of all the presented assumptions we have derived the new model (RHC) for thermal conductivity for nano-fluids:

$$\lambda = \lambda_f \left\{ \frac{\lambda_{pt} + (n-1)\lambda_f - (n-1)\alpha_c (\lambda_f - \lambda_{pt})}{\lambda_{pt} + (n-1)\lambda_f + \alpha_c (\lambda_f - \lambda_{pt})} \right\},$$  \hspace{1cm} (33)

V. RESULTS AND COMPARISON WITH EXPERIMENTAL DATA

In the presented paper we will show analytical computations for the mixture between nanoparticles or nanotubes and reference fluid. The copper and aluminium oxide nanoparticles dispersed in the fluid are very interesting for nanofluid industrial application due to very high thermal conductivity in comparison with copper or alluminum oxides. In our case we have used experimental results from the literature [33] where copper average nanoparticles diameter is smaller than 10 nm. For Al₂O₃ nanoparticles authors Eastman et al. reported [34] that the average diameter is 35nm. The carbon nanotubes have attracted much attention due to of their unique structure and remarkable mechanical and electrical properties [37].

Nanofluids consisting copper nanoparticles directly dispersed in ethylene glycol have been observed to exhibit significantly improved thermal conductivity enhancements compared to nanoparticle-containing fluids of nanofluids containing oxide nanoparticles [34]. The large
improvement in effective thermal conductivity obtained in nanofluids containing metallic nanoparticles presents important potential for revolutionizing industries that are dependent on the performance of heat transfer fluids.

Figure 1 shows the deviation of the results for copper (Cu) in the wide temperature region between the analytical computation between Eliashberg coupling function (E) and Eishah models. The Eishah model is obtained on the basis of comparison between experimental data and empirical equation with optimization procedure with fitting parameters. The agreement between models is satisfactory with maximum relative deviation between both models around 9%. With the CF-1 is presented Eishah model with help of equation (8) and with CF-2 is presented Eishah model on the basis of equation (9). For ethylene glycol (Fig. 2) we have compared our analytical results with the simulations obtained on the web http://www.mhtl.uwaterloo.ca/old/onlinetools/airprop/airprop.html. The comparison shows very good agreement. Figure 3 shows the comparison between analytical calculation and experimental data for aluminium oxide. Figures 4 and 5 show the analytical calculation of mixture between ethylene glycol and copper and aluminium oxide (Al₂O₃) nanoparticles for thermal conductivity ratio. The results for thermal conductivity obtained by X and RHC model show relatively very good agreement in comparison with experimental results. Thermal conductivity predicted by HC model give much lower values as experimental results. Figure 6 shows the thermal conductivity ratio enhancement for the mixture between carbon nanotubes (CNT) and decene (DE). In the Figure 6 is agreement between analytical data obtained by X model and experimental data satisfactory. For the analytical calculation by X model we have used the assumption that the average nanotube is a=500⋅10³ nm long (b=c=12.5 nm). Figure 7
shows the analytical prediction of the mixture obtained by X model between ethylene glycol and carbon nanotubes \((a=50 \times 10^3 \text{ nm}, b=c=12.5 \text{ nm})\).

![Figure 1: Thermal conductivity for copper](image-url)
Figure 2: Thermal conductivity of ethylene glycol

Figure 3: Thermal conductivity of aluminium oxide
Figure 4: Thermal conductivity of mixture copper nanoparticles + ethylene glycol at various composition

Figure 5: Thermal conductivity for Al₂O₃ nanoparticles + ethylene glycol
Figure 6: Thermal conductivity ratio for mixture between decene (DE) and TCNT

Figure 7: Thermal conductivity ratio analytical prediction with X model
VI. CONCLUSION AND SUMMARY

The paper presents the mathematical model for computation of transport properties for nanofluids containing nanoparticles or nanotubes. In the presented paper we have focused on the influence of liquid layer as the reason of thermal conductivity enhancement of nanofluids. With the presented theory is possibly predict all important thermodynamic and transport properties. The analytical results are compared with the experimental data and show relatively good agreement.

VII. REFERENCES


